Philosophy 324A

Philosophy of Logic

2016

Note Twelve

*** There is an instruction-error in online Note #2 on modal axiomatics. At the very beginning, I ask readers to memorize the basic axioms for a given system and go on and memorize the distinguishing axioms of the others. This is the right instruction for modal logic students. It is the WRONG one here. Kindly disregard the instruction Sorry for the error.***

RELEVANCE

1. *Getting started*

Anyone who accepts yesterday's definitions of entailment (or logical implication or its converse, following of necessity from) repeated below, can easily see that *ex falso* straightforwardly follows from them. Suppose we say that $\{A_1, \ldots, A_n\}$ entails B iff it is logically impossible for all the A_i to be true and for B concurrently not to be. Then we have it at once that since it is not logically possible for any proposition A and not-A to be true, it is also logically impossible for the proposition A and not-A and not-B to be true. Hence, by definition, A and not-A entails B.

If, however, we were to ask the neurotypical cognitive being at large whether he thinks that from

(1) Every man is mortal and none actually is

it really does follow, of necessity, that

(2) Vulcan is a real planet

a good many of them would say not. Why not? A common answer is that (1) is about all human beings and (2) is about a (purported) heavenly body. Well, we might say notwithstanding, isn't the fact that (2) has to be true if (1) is, a fact they share in common? Yes, true enough; but what (1) and (2) don't have in common is a common *subject matter.* Nothing that (1) is *about* is what (2) is *about*. In so saying, two ideas emerge:

(3) *Content-overlap relevance.* There is no content which both (1) and (2) share.

(4) *The relevance constraint on entailment.* This precludes the one's entailing the other.

Suppose that we found (3) and (4) worth hanging on to, at least for a while. Suppose further that we wanted to see if we could get these purely intuitive insights into good enough shape for productive work in the context of a rigorously worked out logical theory of entailment and relevance by way of the reconstructive remedies provided by the formalization of these notions. How would we proceed at, say, the elementary level of propositional logic, beginning first with (3)?

Since sameness of propositional content is on its face a *semantic* property, let's first turn to the model theories of e.g. classical propositional logic or any of Lewis' modal ones. How would we formally represent (3)'s concept of content-overlap relevance? Well, the formal sentences of all these systems are entirely without propositional content. The only available candidate for model-theoretic overlap – aside from the fact that (2) is in the model-theoretic closure of (1) – is commonality of the semantic objects T and F. Unfortunately, it takes but 10 nanoseconds before we see the hopelessness of this idea. If it held true, it couldn't be true that F *p* entails a T $p \vee q$, and goodbye to the validity of $\vee$-introduction.

It would appear that the only stab at formally representing content-overlap relevance in any of these logistic systems is to be found among its syntactic features, not its semantic ones. This is precisely the move that Anderson & Belnap made in the early 1960s.

(5) *Content-overlap formalized*: If proposition A is content-relevant to B (in these formal representations) then A and B share some atomic elements. Call this condition **Rel**.

It is striking that **Rel** is not a sufficient condition on content-overlap relevance. Here is why. If we examine every line of the Lewis & Langford proof of *ex falso*, it is easy to see that each of them but the first shares an atomic element with the one from which it follows, and the first shares an atomic element with each of the ones below. So if **Rel** were a sufficient condition of content-overlap relevance, then each line of the proof would be relevant to some line above, and it in turn to a line above, until we get back to line (1). And that would mean that the L & L proof of *ex falso* is home and dry in these systems.

The moral here is that

(6) Sharing atomic content-sharing is *not* sufficient for A & R content-relevance.

The icing on this cake is that there is also a natural language proof of *ex falso*, and nothing in these rationally reconstructive formal upgrades shows that proof to be defective.[1]

The last preliminary point to mention is that the close over *ex falso* between L & L and A & B was motivated by logical realism.[2]

Let's move now to B & R.

2. *B & R relevance*

Greg Restall is a leading figure in the present-day relevant logic research community. When it was time to recruit a first-rate author for one of the volumes of Dov Gabbay's and my eleven-volume *Handbook of the History of Logic*, I asked Greg to take it on, and was delighted when he accepted.[3] It is a long essay of over 100 pages and it displays to considerable advantage Restall's command of this file, as well as his own technically virtuostic contributions.

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[1] Anderson & Belnap have another quite different concept of relevance, in which irrelevance is simply a matter of redundant premisses. This matters for syllogistic and linear consequence, but has no standing in the logistic systems reviewed here. There is no need to bother with it for present purposes.

[2] It is true that Lewis did not exhibit a steadfast realism towards each one of his own systems. He was a realist about S5, but in relation to S2 was a see-what-happens instrumentalist. Let's not forget that Lewis was a Harvard ant that Harvard was the spawning waters of American pragmatism.

[3] Greg Restall, "Relevant and substructural logics", in Dov M. Gabbay and John Woods, editors, *Logic and the Modalities in the Twentieth Century*, volume 7 of their *Handbook of the History of Logic*, pages 289-398, Amsterdam: North-Holland.

When we turn to the book that he and JC Beall have written, we see that the starting point is that given its unmissable ambiguity, there are different and perfectly good concepts of logical consequence. I mean by this concepts of logical consequence in the intuitive sense, and that *relevant* consequence is one of them. Why would they think this? As far as I can see, there is little empirically discernible reason in the linguistics of English to support this claim. So what's going on?

Here is my theory of the case. It is quite commonly agreed by philosophers – I mean philosophers generally speaking – that (as we have seen)

- *Philosophically intuitive consequence:* One statement B is a logical consequence of a set of statements $\{A_1, A_2, \ldots, A_n\}$ just in case it is *logically impossible* for all the A_i to be true and B not.

Certainly, something *like* this is certainly the case. If it *is* so, it matters. The reason why can be set out as follows. It is widely and rightly conceded that, with the help of the negation operator, the concepts of logical possibility and logical *necessity* are interdefinable:

- *Necessity and possibility*: Any logically impossible proposition has a logically impossible negation.

Given the ancient idea that when a statement B is a logical consequence of a set of statements $\{A_1, A_2, \ldots, A_n\}$, the A_i together *necessitate* B, we have

- *The modality of logical consequence*: If it is the case that whenever B is a logical consequence of $\{A_1, \ldots, A_n\}$, it is a consequence necessitated by the A_i together, then we have it that logical consequence is the converse of the *modal* notion of necessitation.

This is an entirely plausible conclusion, and it is clear (to me at least) that B & R are exploiting it to their ambiguation-intent advantage. Perhaps this is inadvertent. Perhaps they've been taken in unawares by the apparently modal character of logical consequence.

Anyhow, here is what happens in the early stages of *Logical Pluralism.* B & R rightly seize on the ambiguity of necessity. In one sense, it means purely *logical* necessity. In another it means *metaphysical* necessity. In yet another it means *mathematical* necessity, and in others still *physical, causal, historical* and *practical* necessity. (B & R don't hit all these buttons; but who's counting?) Given that logical consequence is just necessitation in reverse, why wouldn't it follow that

- *The ambiguity of consequence*: The concept of logical consequence is itself ambiguous in virtue of having been definitionally imbued by the undoubted ambiguity of the necessitation relation of which logical consequence is its converse?

How does one respond to this rather formidable-looking chain of reasoning? I respond to it as follows. (You might not. If so, prepare your reasons why.)

- The *cited* and *real* ambiguities of necessity make no credible claim to the ambiguity of *logical* necessity. That some things are practically necessary doesn't mean that this in any way matters for logical necessity. The cited ambiguities do nothing to ambiguate the property of logical necessity.

- Even if the concept of logical necessity in English were ambiguous, that would fall short of showing that one of its several meanings is *relevant* necessity, hence *relevant* consequence.

Consider the concept of relevance in English. Is there in established English usage any foundation for the view that one of the meanings of *relevance* is "relevance by logical necessity alone"? I respectfully suggest that the answer is "no". If I'm right, the ambiguity of necessity makes no case at all for the existence of a relation of relevant consequence in English. So, once again, what's going on?

- Here is my stab at answering this question. In *all* formal systems, *all* model-theoretic properties are relativized to mathematical structures called *interpretations*. What we have here is another case of the tort that Tarski committed upon semantics. Scan any natural language and we'll find nothing there that means "representation" in the sense in which that expropriated English word is made to mean in model theory.

- All the same, *one* of the meanings of "representation" in English is "meanings". When people steal a settled word for entirely different referential purposes, they'd do well to keep it in mind that stolen words tend to retain their usual connotations. So what the robbers are doing is radically changing the *denotation* of "interpretation" and doing much less well in changing its usual *connotation*.

  The result? When B & R say that one of the *meanings* of "logical implication" is relevant logical implication", they no doubt inadvertently equivocate on "meaning" as between meanings as model-theoretic interpretations and meanings in the mother tongues of humanity.

- All that remains to say specifically about relevance will be said (or have been said) in Thursday's class on October 13th.