

Philosophy 324A

Philosophy of Logic

2016

Note Twenty-three

THE TRICHOTOMY

1. It might strike you as odd that in a course on the philosophy of logic we'd go three long months without ever asking head-on what it is that takes for a mode of enquiry to *be* logic, and what logic is *about*.
2. In this subject's nearly two-millenia history these questions had a ready and fairly constant answer:
 - Logic is the systematic study of three relations between premisses and consequences:
 - * consequences the premisses *have*
 - * *spotting* the consequences they have
 - * *drawing* the consequences one spots.

Using logic's founder as our reference point, whether S' is a consequence of S has everything to do with S and S' and nothing to do with people. However, both spotting and drawing are events in which people are directly implicated.

3. For Aristotle, people is where the action is in logic. But to say what he wants to say about people, he'll have to lay down some foundational principles governing the relations between the propositions of which people take note when the spot and/or draw the consequences that propositions have when they serve as premisses of deductive reasoning.

Aristotle deals first with the non-people part of logic, then moves to the people part, beginning with drawing and then moving on to spotting. Note well that the people part of Aristotle's logic is amply stocked with *empirical* representations of people and their doings when they spot and draw. Apart from the decision to schematize statement-expressing sentences of *Greek*, there are no empirical representations in the having-part of this non-people logic.
4. Let's now leap ahead to the latter half of the 19th of our era. To the people who mattered in this period, the inherited logic was of little help to them in their desire to stabilize modern mathematics. They thought the same of the empirically sensitive logic of J. S. Mill (1843). Peirce said something crucial when he proclaimed that logic, in the strict sense he was in process of helping to invent, has nothing to do with how people think. What's crucial about this development is the expulsion of people from logic.
5. What's also rather maddening about it is that, contrary to what we might have expected, when

the new logic came into full flower, the having/spotting/drawing trichotomy was still left standing. Drawing is catered for by rules of inference. Spotting is catered for by decision methods. Having is catered for twice-over, once semantically and otherwise syntactically (by model theory for ε , and by proof theory for \vdash).

6. Not surprisingly, questions and difficulties arise.

- One which we've spent a lot of time on is whether, and if so how, provisions for having, spotting and drawing in purely formal and mathematical environments can carry the water for having, spotting and drawing in natural language environments. And whether it matters one way or another whether it does or doesn't.
- Another, which we only slightly discussed, is the question of how these mathematicised logics of non-people conceive of how its inference rules would be implemented, and how its decision procedures would be run. What would be the point of having *unimplementable* rules of inference and *unimplementable* decision procedures?

The answer to this lies in the largely unspoken assumption of implementability by *ideally rational* reasoners, an assumption that takes on a full-voiced axiomatic presence in standard theories of belief-change and rational decision theory.

The problem with this is twofold.

(i) How is it known to be axiomatic (or even true) that this is how a rational agent would draw his inferences and run his decision checks if he were ideally rational? How, for example, is it known that the ideally rational agent is a being who has perfect information, knows all the logical truths, and closes its beliefs under consequence?

(ii) If it is said that this is what would be in the ideally rational reasoner's nature to do, how does it come to be the case that what it is in his nature to do is normatively authoritative for what *we* do, we the *people*? Does anyone in his right mind really think that the more consequences we draw of any belief we hold, the better? If that were so, wouldn't we devote ourselves to endless iterations of the or-introduction rule?

There is no space to litigate these matters further. So I'll conclude by saying that some of the most dazzling defensive lawyering you'll ever see in a non-legal context is on offer in the precincts of ideal rationality.

7. The last matter there's space for in this final note could be seen as resource-saving issue, a matter of cognitive economics. In Aristotle's case, there are three different *sub logics* – one for having, another for drawing, and a third for spotting. In this the mathematical era, most of the investment is in spotting and drawing the theorems of the logic on hand. The having part requires less labour and is usually settled early on by the definition of having under the regulatory control of closure-conditions. At this point, the nose of the economy-minded logician starts to tweak. "Wouldn't it be lovely if in providing for having, we did all the heavy work required for spotting and drawing?"

It is upon this twitching of the economic nose that the burden of most of the wrangling about *ex falso* falls. If anything is clear, it is that *ex falso* is false for drawing. Why not, then, make it false for *having*? The trouble is, there's a world of difference between having and drawing. It is a plain fact that having is not resistant to *ex falso* in anything, if at all, like the way that drawing is.

But, needless to say, that won't deter anyone who thinks that there are no world-facts of the matter about logic from taking the position that we are free to make of having anything we please. My only reply to this is, "Small wonder that in this the heyday of deviance (Quine's term for it) logicians have been doing such a jolly good job in talking themselves out of gainful employment."¹ Verily, it is the vale of tears.

¹ What Quine means by deviant logic is logic in its nonclassical variations. See *Philosophy of Logic*, chapter 6.