Philosophy 324A

Philosophy of Logic

2016

Note Three

The Mathematical Character of Logic

1. **The formal turn**

- *From what*? Properties expressible in natural language

- *To what*? Properties definable for artificial formal languages.

Why?

- Meanings of NL terms are vague and ambitious, which impedes the precision and rigour of such languages, and calls into question their foundational stability.

- The desired rigour and precision required for logic will have to be found elsewhere.

Where?

- In systems such as those described in notes 1 and 2 on the course webpage.

Why are these appropriate places to move to?

- *Answer (1):* Because all the trouble stems from NL meanings, and there are no NL meanings in these artificial languages.

- *Answer (2)*: Same as above, *plus:* (i) However, the logical terms of these formal languages do have meanings of a purpose-built kind (model theory), which aren't meanings in the NL sense of the term. Artificial languages require artificial meanings. (ii) Same as answer (1), but contrary to (i), Artificial languages require meanings of no kind. Their systems can be managed entirely by syntactic considerations. (Proof theory).

Why formal?

- Because (see point (ii) just above), all properties of interest can be defined in such a way that the linguistic items that have those properties do so solely in virtue of their respective logical forms.

2. **The alienation question**

All properties of logical interest are defined in NL and instantiated by NL constructions. People draw their inferences in NL and advance their arguments in the same way. Even number theory is worked up in NL. So when logicians decide to abandon natural languages for artificial ones, don't they risk *alienating* the properties they artificially define from the properties of the languages in which human beings conduct their logical affairs?

The idea here is that when we change the subject from X (NL-instantiated properties) to Y (FL-instantiated ones) sometimes we arrive at a better understanding of X. But, given the radicalness of the difference between NL-properties and "counterpart" FL-properties, isn't this a forlorn hope?

> **Reply A**: The *model theory* of a logistic system is carefully designed to *mimic* the NL properties. It is intended to represent them in illuminating ways or, in other words, to model them formally. In this way, alienation turns into clarification.
>
> **Reply B**: The *proof theory* of a logistic has no capacity to generate properties that simulate NL properties such as logical truth, logical entailment, and logical consistency. However, a logistic systm's *metatheory* is capable of showing that its proof theory is equivalent to its model theory, establishing this by completeness and soundness metaproofs for theorems and logical truths, for deducibility and entailment, and for "syntactic" consistency and "semantic" consistency. Therefore, if model theoretic properties really do clarify natural language counterparts, so do their proof theoretic ones at, shall we say, one further remove.

3**. The mathematical turn**

> *Why*?
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> *Answer (1)*: Since the purpose of logic is to provide an intellectually stable home for arithmetic; and in so doing to give to arithmetic the foundational security it's unable to give itself, the *subject matter* of logic gives it an inherently mathematical character.
>
> *Answer (2)*: Moreover, the technical machinery that drives a logistic system employs mathematical entities such as sets and functions, and employs methods such as mathematical induction and definition by recursion.
>
> *Answer (3)*: What is more, it takes considerable mathematical virtuosity to build the contrivances that do the business of mathematical logic. It only stands to reason that a substantial part of logic is the mathematical examination of its own methods, the effect of which is the mathematicization of the *metatheory* of logic.

Reply C: Contrary to replies A and B, the sole role of logic is to produce proof theories whose sole function is to aid mathematicians to negotiate from well-accepted mathematical inputs to well-accepted outcomes, without *any* need to raise questions as to how proof procedures are to be interpreted. (Hard-nosed formalism)