

Philosophy 324A
Philosophy of Logic
2016
Note Thirteen
PLURALISM

B&R pluralism

As we have it now, the bedrock of pluralism in the B&R sense is embedded in GTT:

*An argument is valid_x iff in every case_x in which the premises are true, so is the conclusion.*¹

Note well that the subscript x is employed with ambiguating intent.

- **Ambiguity of “valid”:** *Given that the concept of case is ambiguous, it straightforwardly follows that the concept of validity also is.*

Accordingly,

- **Ambiguity of “logical consequence”:** (“Restricted”) *TT provides that the concept of logical consequence is also ambiguous.*

From which we have

- **The genus-species thesis:** *Logical consequence, which is the centre-piece of logic, is not just one relation unto itself. It is a relation – the mother ship – of different but perfectly genuine species of logical consequence, each of which has a perfectly genuine logic that caters to its peculiarities, without escaping the reach of either GTT or TT.*

B&R go on to say that there at least *two general types* of admissible theoretical workings-up of these logics, one of which does well in certain theoretical contexts, and other doing well in various others. Concerning which they write at page 48:

- **B&R pluralism:** *Our logical pluralism recognizes at least admissible instances of GTT. In this chapter [= chapter 4] we have found pluralism: possible worlds and Tarskian models both yield admissible precisifications of ‘follows from’, the NTP [necessary truth-preservation] and TM [Tarski model] accounts.*

¹ Recall, B&R *cases* are either model-theoretic interpretations or possible worlds. Interpretations here aren’t meanings, however, and worlds aren’t worlds. My, my!

- **TM-logical consequence** is logical consequence in virtue of logical form alone. **NTP-logical consequence** allows for one thing to follow from another owing to a connection between antecedent and consequent which, although necessary, is not captured in a purely syntactic way. E.g. “The shirt is coloured” is a logical consequence of “The shirt is red”.

B&R then proceed to advance

- **The favoured trio thesis:** *At least three logics make the NTP or TM cut: the logic of classical consequence, the logic of intuitionist consequence and the logic of relevant consequence* (of which *the logic of paraconsistent consequence* is a prominent variation.)

Notice here that B&R pluralism has and is designed to have a **realist** cast.

- **Realist intentions:** *There really are different but perfectly genuine relations of logical consequence.*
- **Corollary:** *Correspondingly, there are different but genuinely good logics that really do account for these differences in **objectively accurate** ways.*

However, B&R pluralism also has an accompanying **instrumental** cast.

- **Instrumentalist rationales:** A criterion of admittance into the B&R family of honest-to-goodness logical consequences is what a candidate for membership is actually **good for**.

For example,

- **Classical consequence:** *Classical consequence (TM) does well as the consequence relation for classical mathematics* (p. 48)

whereas

- **Relevant consequence:** *Relevant consequence (NTP) does better as the consequence relation of inconsistent (also incomplete) theories, and also for theoretical treatments of the modalities.* (pp. 58-59)

Some observations

- It may be of some interest that the B&R pluralist-legitimacy net manages to catch all the already well-established logics – classical, modal, intuitionist/constructivist, and relevantist/paraconsistentist and these dialethic variations.
- If we just look at the plethora of well-made, technically virtuosic logics currently (and persistently) making the scene in the journals, it begins to look as though B&R might

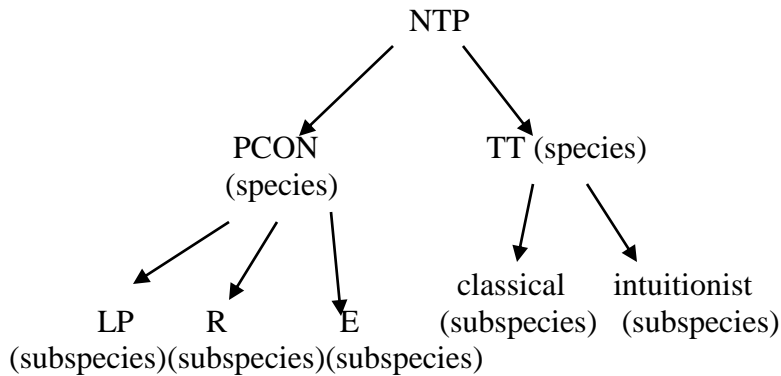
have fashioned a rather cherry-picking kind of pluralist legitimacy. It is open to question whether any of the following logics make the B&R cut: linear logic, dialogue logics, public announcement logics, attack-and-defend logics, quantum logics and game-theoretic logics.

- On the other hand, *Logical Pluralism* is quite a small book with lots of interesting things to say of the matters it covers. Perhaps the right reading of its intention is to see the criteria which look fondly on the logics it cites as a template for the recognition of the many other logics that fit its specifications. In that case, B&R pluralism would be a *paradigm* of *all* the logics that lie within its criterial embrace. If so, have we any sure-handed idea of how far this net casts and of the logics to be found in its catch? Even assuming a satisfying answer to this question (which we don't get in B&R), other questions press for attention.
- **The genus-species puzzle:** *If classical and relevant consequence are different species of the logical consequence relation, what is their genus?*
- It can hardly be found in GTT, since it gives (or purports to) the genus of *validity*.
- True, we can get to consequence by the way of the restriction on GTT that gives us TT. This looks promising, since it is *logical consequence* that is the true centre-piece of logic.
- However, as we see, classical and intuitionist consequence instantiate TT, but relevant consequence doesn't. It instantiates NTP, which captures a *broader* notion of consequence than the one captured by TT.
- Recall, NTP allows for sentential connections whose necessity is not captured in a purely syntactic way. E.g., that the shirt is red logically implies that it is coloured.

Solving the puzzle?

- B&R *might* want to say that actually NTP and TT are themselves two different *genera* of logical consequence, and that relevant consequence is one of the NTP species (as are many systems of paraconsistent consequence), whereas classical and intuitionist consequence are species of the TT genus.
- In which case, is there a **super-genus** of which NTP and TT are themselves species, though each a genus itself? If there is, B&R doesn't specify it.
- On the other hand, they might prefer saying that NTP is the one and only genus, of which TT is but one of its species.

In which case, we would have the following taxonomy for logical consequence.



[“PCON” is my made-up general-purpose name for paraconsistent logics. “LP” is the name for the basic Priest system of dialethic – or paradoxical – logic. “R” names one of the key A&B systems of relevant logic and “E” another of them developed by Bob Meyer.]

We come now to what I’ll call the *boot-strapping question*.

- All the standard work known to me on the genus-species relation requires that when Y is a species of X, it follows of (some kind of) necessity that any Y-thing is implied to be an X-thing, and that a species is distinguished from its genus by *differentiae* that characterize the species but not the genus as such. Since we’re here talking about *deductive* necessitation, we would appear to have it that the relation borne by a genus to a species is *itself* one of logical necessitation.

Which triggers a rather basic question:

The genus-species question: Which one of the different consequence relations in the B&R taxonomy undergirds the necessitation-relation within?

- Is there a **bootstrapping problem** here? Is the B&R taxonomy up to its ears in **circularity**?

Something to think about

B&R pluralism is strictly about *deductive* consequence relations. Much of the more interesting work in logic these past forty to fifty years (and the source of abundant multiplicities of its own) has been devoted to *nonmonotonic* consequence relations – *defeasible* consequence, *default* consequence, *autoepistemic* consequence, and so on. These developments, largely at the prodding of computer scientists and AI specialists, are rooted in a desire to simulate the realities of *human reasoning*, which was the founding mission of logic at its very beginning, notwithstanding that the relation of following of necessity from is of core importance but limited, but not overridingly so.² The reason why is that most of a human being’s premiss-conclusion reasoning even at its best is not abductively correct.

² Even so, except for certain autoepistemic logics, all nonmonotonic ones display nonmonotonic consequence relations at about two removes from monotonic ones. The moral? Most nonmonotonic logics operate in the

- Of course, it hardly needs mentioning that all the systems of *inductive logic* are anchored in what purport to be non-deductive (hence nonmonotonic) consequence relations.

A challenge

What might we think of the following line of thought?

- **Pluralism rests upon a mistake:** *The species-view of logical consequence rests upon a disabling misconception about the differentiae that drive the genus-species relation.*

Let's take logic's foundational theory as our test case, Aristotle's logic of syllogisms. Aristotle has a twofold notion of syllogisms. The foundational notion is that of the **syllogism as-such**.

- **As-such:** A syllogism as-such is a *valid* argument made up of an ordered triple of propositions, whose terminal member is its conclusion, and the other two are its premisses, subject to the following further conditions:
 - (a) a *proposition*, in Aristotle's special employment of the term, must be a natural-language statement in one of the following forms: "All A are B", "No A are B", "Some A are B", and "Some A are not B".
 - (b) there are two premisses *only*
 - (c) there are no *redundant* premisses
 - (d) premisses may include no terms "from the outside", in other words, *no off-topic* premisses.
 - (e) Each premiss must be *internally consistent* and the two of them *jointly* so.
 - (f) Conclusions must be limited to some *one* proposition *only*.

In virtue of condition (b), syllogisms are nonmonotonic, and in virtue of condition (c), they are *full-use relevant* in something close to one of the A&B meanings of relevance. In virtue of (d), syllogisms approximate to A&B *atom-sharing relevance*, and in virtue of (e) they are a limiting case of *paraconsistent*. Finally, in virtue of (f), syllogisms have an unmissable *intuitionist* cast.

Imagine that! At its birth, *Aristotle's* logic is nonmonotonic, relevant, paraconsistent and intuitionist, but its underlying validity condition has none of these properties. As we have them now, syllogisms-as-such just hang about in logical space, requiring for their existence no human effort beyond the creation of natural languages capable of expressing propositions of these four sorts.

In time, propositions of these four forms came to be known as *categorical* propositions. Aristotle's restriction of syllogisms to constructions of categorical propositions is explained in a way that is almost certainly mistaken, yet reflect great credit upon his logician's instincts. In *On Interpretation* at 17^a 13, 18^a 19-24, Aristotle launches what might be logic's first *reduction thesis*. He says, without proof, that everything stateable in any human language is re-stateable without relevant loss in the language of categorical propositions. If this were true, it would

gravitational field in which deductive consequence is the mother star. See here David Makinson's excellent student text, *Bridges From Classical to Nonmonotonic Logic*, Texts in Computing, London: College Publications, 2005.

greatly simplify the semantics and grammar of the language. All that would be required for, say, English is a theory for statements instantiating one or other of these four categorical forms.

We come now to what may be logic's *second* reduction thesis which, although almost certainly is untrue, once again reveal Aristotle's acute perception of the economics of reducibility, in which problems are framed for the big, yet wholly solved for the small. This second thesis says that all deductively correct reasoning in English can be re-engineered as correct *sylogistic* reasoning. Since syllogisms are just three lines long, effecting thereby enormous economies for the logic of deductive reasoning.³

Finally, the concepts of argument-validity and the corresponding one of logical consequence are not dealt with in Aristotle's logic. They are **undefined primitives**. Why would this be? It would be because for his present purposes, Aristotle has no theoretical interest in the valid arguments as such. His interest here is reserved for the different kinds of valid arguments you get when you impose eligibility conditions on **premises** and/or **conclusions** of arguments that are already acknowledged to be valid.

Syllogisms in-use are syllogisms as-such carved out by further restrictions on premisses and conclusions. For example, an as-such syllogism when used as a **refutation-argument** must be a syllogism-as such that satisfies certain additional conditions. E.g., premisses are restricted to answers to yes-no questions put in attack-and-defend exchanges between two antagonists. The constraint on conclusions is that they must be the *contradictories* of the answerer's original thesis. On the other hand, if a core syllogism is being used as a **scientific demonstration**, its premisses must be in the deductive closure of science's *first principles* (or axioms), whereupon its conclusion must also be in that same closure (that's the *conclusion condition*).

The key question

- We see that the differences between syllogisms-as such and syllogisms-in use is entirely a matter of constraints on *admissible premisses*, and/or conclusions of arguments in which those conclusions already follow from those premisses, and leaves entirely unmolested the role of deductive validity. Why, then, would we think that the very clear differences between refutation-arguments and scientific demonstration reflect *any* difference in the underlying deductive consequence relation?
- It is the same way with the very clear difference between syllogistic as-such and validity. Even at the as-such level, all the difference is a matter of constraints on premiss-eligibility and/or constraints on how conclusions are structured, not on how they are validly derived.

Why, then, would we suppose that *these* differences drive a distinction between different meanings of *deductive validity*?

Now what?

³ There are other reduction theses, principally in *Prior Analytics*, where they receive convincing proof. These not detain us here.

Turn now to all the purported species of the logical consequence relation which make the cut of B&R logical consequence. Check to see whether their respective *differentiae* cannot be found in conditions which govern premiss/conclusion admissibility. I leave this as an exercise.