

Philosophy 324A

Philosophy of Logic

2016

Note Fourteen

LOGIC'S SUBJECT-MATTERS

1. Some foundational confusions

Russell once quipped that logic is the one discipline which isn't about anything and whose practitioners don't know what they are talking about. Of course, this is right only on a largely irrelevant technicality, namely, that a logic's domain of quantification is restricted to unidentified entities – the values of the variables bound by “ \forall ” and “ \exists ” in an unspecified domain of individuals. In that limited sense, pure quantification theory lacks a subject-matter. But in a much more central sense, the logics that Frege and Russell were working on before 1902 certainly did have a subject matter – the denotata of “all”, “there exists at least one” and “set”. (Don't forget, Frege and Russell thought that set theory was a proper part of pure logic.) For brevity, let's say:

- Quantification and sets are part of the *subject-matter* of Frege's and Russell's logics.

However, it would be quite wrong to think that these are all that their logics were about.

- They were also purpose-built to tell us something philosophically important about *arithmetic*. Namely, that every truth of number theory can be re-expressed without relevant loss as a theorem of the pure logic of quantification and sets.

It is now easy to see a distinction between

- What we *want* to know and what we *need* to know in order to get there.
- Frege and Russell weren't bursting to know the truth about quantifiers and sets, or even about number theory, of which they already knew pretty much what the Peano axioms covered in 1889.
- What they were bursting to know was how to effect the reduction of arithmetic to pure logic – that is, whether and how *logicism* works.

From this a helpful distinction pops out. For Frege and Russell,

- Logicism was the *target* subject-matter of logic, whereas quantification and sets were its *means-ends* subject matter.

It is well worth knowing that logicism doesn't make the indexical cut in *Logical Pluralism*, wherein logical consequence is the heart of logic. B&R don't get around to telling us what

“at the heart of” means in this context. Fair enough, it is a well-travelled expression subject to helpful contextual disambiguation. Well, then, what does it mean in *this* context? Does it mean subject-matter, and if so, what kind – means-end or target subject-matter? Before answering this, let’s briefly return to Frege and Russell, and ask how logical consequence enters their picture. Their axioms, formation and transformation rules are set out as conditional statements, subject to the expectation (which isn’t fulfilled in lots of other systems) that if the conditionals are true, their respective antecedents *logically imply* their respective consequents.¹ Various other notions are no less important – e.g. negation, conjunction, disjunction and our old friends the quantifiers. Unless provision is made for how these work, the logic won’t deliver the goods for logicism. Therefore, in the Frege-Russell context, conditionality/consequence is a *means-end* subject matter, and no more central a concept than negation of existential quantification.

Let’s now swing back to Aristotle. We can say the same sort of thing about his logic. Clearly, syllogisms are a goodly part of its subject-matter. But when we ask “What explains Aristotle’s interest in this subject matter”, we get quite a complex and interesting answer. Aristotle tried (unsuccessfully) to show that every kind of good deductive reasoning/argument is reducible without relevant loss to syllogistic reasoning/argument. As mentioned earlier, Aristotle also thought that everything stateable in NL can in this same sense be said in the language of categorical propositions, of which there are just four kinds. By their very construction, syllogisms have a very slight and easily managed structure. If you get the logic of syllogisms right, you’ve got the entirety of deductive reasoning/argument right as well. Without these reduction relations, this would have been far too big a task not only for any given theory to get right, but to get right in ways that are *intelligible* to the reasoner/arguer at large. (Note that argument and reasoning are also significant parts of the subject-matter of this logic.)

In *Prior Analytics*, Aristotle achieved the overriding objective of his enterprise. He proved that any piece of valid deductive reasoning and any valid argument is *recognizably so* to the neurotypical thinker/arguer, using methods that take little time to apply, are easy to apply, and are self-evidently the correct ones. The proof – called “the perfectability proof” – is nearly perfect (no pun) and is easily repairable (once you know how). This is getting to be quite a lot of subject-matter. But there is little doubt that the production of something quite like a *decision-procedure* for all of deductive reasoning/argument is the *target* subject-matter, whereas all the others are means-ends ones. One, to be sure, is validity/logical implication. But, as we saw, Aristotle’s logic has nothing to say of it, beyond the fact that syllogisms always have it.

Come back now to the B&R version of classical logic. If we ask “What is this logic about?”, the answers come tumbling in. Classical logic is about atomic and molecular formulae, about negation, conjunction, disjunction, conditionality and biconditionality, about quantification and truth-values, about logical truth, logical implication, logical consistency, about interpretations and models, and on and on. “No”, comes the reply, “you’re not understanding my question. What I want to know is what all this stuff is *for!*” How should we answer this? Quine has a long-held answer: Classical logic serves as the template for the formulation of scientific theories in a philosophically secure manner.

¹ It is true that *Principia* drew some heavy fire for trying to make do with the *material* conditional and *material* implication. But this needn’t deter us here, although I’ll come back to it in the section to follow.

In *Logical Pluralism*, we cut straight to the chase. It's not framed in terms of subject-matter or aboutness, but rather in terms of the "at the heart of" metaphor, leaving it unclear whether this heart has room for other coequal presences, e.g. logical truth. Moreover someone might raise the following objection.

"I agree that conditionality is the work-horse of logical theory. Virtually everything we say in individuating all these other subject-matters is said in conditional or biconditional terms. E.g. "A takes T iff $\sim A$ takes F, for all A". Suppose we decided to dignify this work-horse status by calling conditionality the heart of logic. Since consequence goes hand-in-hand with conditionality and conditionality here is logical, also at the heart of logic we find logical consequence. So far so good, but there might be a problem here. Lots and logs of logicians deal with the formal language of classical logic, where "if ... then" and "if and only if" lie at its heart, without saying a word about logical consequence, preferring so speak of logical deducibility instead. That's a big difference. Logical consequence is a *semantic* notion, whereas logical deducibility is a wholly *syntactic* one. So the case for the centrality of the former seems to lack legs."

I suppose that there are at least a couple of rejoinders that B&R could consider. One says that classical logic is an *inherently* model-theoretic one, and therefore that proof-theoretic treatments of that same formal logic aren't classical. Perhaps the shortest answer to this rejoinder is "Just who is kidding whom?" A second response is a bit more B&R-esque. It goes as follows.

- There is a *generic* notion of following of necessity from, of which model-theoretic logical consequence and proof-theoretic logical deducibility are two different but perfectly legitimate *species*. So it is quite true to say that this generic notion of following of necessity from is indeed the heart of logic.

One way of testing this (rather artful) move is by shifting our gaze to modal logic, beginning with its early modern stirring in the first decade of the century past.

2. Modal logic

** Before getting started, here is notice of some further mandatory reading, a paper on my webpage under the heading "Drafts and pre-prints". The paper is entitled "MacColl's elusive pluralism". It appears in Amirouche Moktefi and Stephen Read, editors, *Hugh MacColl After One Hundred Years*, pages 205-234, Paris: Éditions Kimé, 2011; a guest-edited number of *Philosophia Scientiae*, 15 (2011). As with "Does changing the subject from A to B really enlarge our understanding of A?", the MacColl paper must be read, but much of it will not be subject to examination. Only those places where the paper bears directly on the matters discussed here is there examinable material.**

MacColl is an overlooked figure of importance to present-day logic. He is the early anticipator of Lewis' logics of strict implication, and of relevant and paraconsistent approaches and also, some say, of logical pluralism. As mentioned in class, he tangled with Russell over whether material implication could serve as the all-purpose implication relation for logic. We'll come back to this just below. Before going there let's frame a simple question about modal logic in the manner of Lewis' axioms. Suppose we asked the logician in the street, "What concept or

concepts lie at the heart of modal logic?” what do you think that her answer would be? Can there be any doubt that she would say

- *The heart of modal logic*: At the heart of modal logic are the concepts of *logical necessity* and *logical possibility*”.

Okay, now back to the fight over material implication. As you’ll recall from your first contact with the propositional calculus, the material implication relation draws the well-known charge of *paradox*. Every F-taking sentence materially implies every other sentence (including its own negation), and every T-taking sentence is materially implied by every sentence whatever (including *its* own negation).

One answer to this charge is easily summarized: Of course, material implication can’t do the heavy lifting for even propositional logic. However, when harnessed to the model theory of propositional logic, we get what we need. Any *tautological* material conditional statement will do all the heavy-lifting of logical implication. The point to notice is that we don’t need to clutter up the vocabulary of propositional logic with a redundant symbol for logical conditionality. The \supset -sign is perfectly adequate, because A logically implies B just when “ $A \supset B$ ” is a tautology.

Still, MacColl appears to have favoured a more cards-on-the-table approach. Let’s insert into the working vocabulary of the object language of the propositional calculus a *purpose-built* symbol for logical conditionality or – as Lewis would call it “strict” conditionality – expressed by the new symbol “ $\text{—}\mathfrak{3}$ ” With it comes *strict implication*. Whenever $A \text{—}\mathfrak{3} B$, A strictly implies B. Strict implication is subject to “paradoxes” of its own, one of which is *ex falso quodlibet*.

- Any logically impossible sentence strictly implies any sentence whatever (including its own negation).

The other is *ex falso*’s flipside (nearly enough)

- *Verum ad quodlibet*: Every logically necessary sentence is strictly implied by any sentence whatever (including *its* own negation.)

Here is a Lewis & Langford-*like* proof:

(1) A	by assumption
(2) $(A \vee B) \wedge (A \vee \sim B)$	1, tautological consequence
(3) $A \wedge (B \vee \sim B)$	2, distributivity
(4) $B \vee \sim B$	3, n-elim

I leave it as an exercise to “Woosieize” this proof for English.

The question now is what is to be made of these paradoxes? Lewis would later say that they aren’t paradoxes at all, and that strict implication is the very same relation as entailment for natural language.

MacColl wasn’t so sure, and there is reason to think that MacColl suggested both the relevantist and paraconsistentist options. And some scholars think that he was also sympathetic to a pluralist approach to logical implication, wherein the paradoxes would be *true* of one kind of

logical implication and yet would fail for another kind of equally genuine logical implication relation. Some say that all these options were originated by MacColl. If so, MacColl is the tragic wee man overlooked by the pompous ignoramuses of later generations.²

What, you may be asking, is the point of this historical diversion? Weren't we supposed to be discussing the logics of necessity and possibility? Yes, of course, and we'll get back to that now. But the diversion hasn't been pointless, not in the least so. Why? Because a large part of what absorbed MacColl and later Lewis was precisely the logical consequence relation, and what would be the best way of treating it. At this stage in logic's modal history, point to B&R! Consequence does indeed lie at the heart of it all, *at that juncture*.

We've now arrived at another point of transition. By 1912, Lewis believes that the consequence question is now settled. We know what honest-to-goodness consequence is. It is strict consequence. When we get to his five S-systems, strict implication is a given. What matters now is how logic is to be reconfigured for the theoretical plumbing of the modalities that occur in logical implication's entirely correct defining conditions, namely, the *necessity* modality and the *possibility* modality:

- B is a logical consequence of A iff it is logically impossible that A and not-B.

or equivalently,

- S logically implies B iff B follows, of necessity, from A.

What Lewis wants to do is to formulate a logic for these modalities, in fact, four variants of the first one, namely, S1.

It is now wholly explicable that, in reply to our earlier question "What concepts lie at the heart of modal logic?", the answer would be the concepts of necessity and possibility. The reason why is that this is precisely the *right* answer.

Let's turn now to Lewis (implicit) pluralism, concerning which he is of two minds. The two minds that Lewis is in at this point is a combination of *before-the-fact realism* and *after-the-fact irrealism*. Lewis doesn't discuss these matters directly. So it is left to those of who've followed him to speculate. I speculate as follows:

- *Lewis' realism*: There are at least two different concepts of *bona fide* logical necessity and logical possibility. They are captured formally by S5 and S4.
- *Lewis' irrealism*: There are at least three different *made-up* concepts of logical necessity and logical possibility, formalized by S1, S2 and S3 respectively.

Concerning the first-bulleted speculation, John Burgess is helpful.

² I don't know how wee he literally was, but in the metaphorical sense of being wholly overlooked he was indeed wee, never mind his papers in *Mind* in 1880 and 1908 and a book from a reputable London publisher, also in 1908. MacColl held no academic appointment. He was a Scottish school master teaching in Boulogne-sur-mer in northern France.

- “In fact, to the extent that there is any conventional wisdom about [it], it is that S5 is correct for the alethic logical modality [= truth by virtue of logical form], and S4 correct for the apodictic logical modality [= demonstrability by pure logic].”

Burgess goes on to say:

- “However, nothing said so far constitutes even an informal ‘proof’ that no formula not a theorem of S4 is correct for apodictic logical modality, or that no formula not a theorem of S5 is not correct for alethic modality. And indeed there is no generally accepted informed argument for the first claim, though a convincing one can be given for the second claim.”³

Let me add that, so far as I know, there is nothing in the record to say that Burgess has caught Lewis’ own S5-S4 difference here.

Let’s bring this note to a finish with three concluding observations:

- The apparent conflict between Lewis’s realism and irrealism might be explained away by his *pragmatism*, which echoes Russell’s own after 1902. For Russell, realism is fine until it can’t give you what you absolutely must have for the unfinished business that lies ahead. When it comes to that, the only thing to do is make up the concepts that will ease the unfinished journey.
- Lewis is a different kind of pragmatist, and one might say a less instrumentally driven one. Where Russell authorizes made-up stuff for the demands that now call for them, Lewis is prepared to make stuff up simply to have new concepts on hand in case the *need* for them ever arises. (Think here of Riemann’s geometry.)
- Finally, no symphony can be written for logical pluralism in the absence of an understanding of its historical antecedents. From that proper perspective, pluralism is a big sprawly thing, and very much a movable feast.

Post scriptum: Modal logic, whether the alethic, epistemic, doxastic, deontic, tensed or temporal ones, doesn’t make the indexical cut in *Logical Pluralism*.

³ John P. Burgess, *Philosophical Logic*, Princeton: Princeton University Press, 2009; p. 65.