Philosophy 324A
Philosophy of Logic
2016

CORRECTION TO NOTE #4: In section 5 on functions, please re-write the second-last bulleted sentence by replacing "another system" with "itself".

Note Five

FUNCTIONS AGAIN

1. We can think of a function as a certain kind of abstraction from the *translation* relation.

2. The abstraction omits or discounts the following:
   a. the language-to-language factor
   b. the meaning factor
   c. kind restrictions
   d. property-preservation requirements, (truth, meaning, structure, etc.)

One or more of these conditions can be fulfilled in various cases. Think here of the function that maps the natural numbers to the even natural numbers. However, none of these conditions is required in every case.

3. Functions are usually expected to satisfy some positive conditions too.

- Domains = (= sets of *arguments*) and ranges (= sets of *values*) must be well-individuated and have precise cardinalities.

4. Functions operate as search engines, many of them algorithmically so. For each of the arguments in its domain, a function finds its one and only mate in the range of its values.

5. An atomic valuation of a logistic system is a function v assigning to each of its atomic formal sentences exactly one of the undefined objects T, F as its value. Atomic valuations are one-one relations.

6. A *formal representation* in CQT of a fragment of English establishes an *asymmetric* one-to-one correspondence between any sentence of CQT and its unique "image" or value in a proper subset of English.

7. Note: Translation into CQT is the *converse* of the formal representation relation.

8. Formal representations of this sort are subject to certain conditions.

- The representation relation between sentences decomposes into representation relations between such subsentential components as the formal sentences may have – e.g. between formal names and English names, formal predicates and English ones, and so on.

- when a formal sentence, e.g. "F(a)" is a formal representation of e.g. "Mary is brilliant", it is normally expected that if $\mathcal{V}$(F(a)) = T, then "Mary is brilliant" is true.
- This is the only sense in which formal representations of this sort are truth-preserving. What actually happens is that the representation maps in such a way that if "F(a)" has T as its value, "Mary is brilliant" satisfies the predicate "is true".
- There are different kinds of case, notably the many-valued representations of vague sentences of English. In such cases, the representation is not in always truth-preserving, but rather sententially by precisifying. For example, "It is not quite daybreak yet" might map to a formal sentence with no representative of "quite", whose value is some intermediate "designated" object I from a range of values {T, I, I′, F}
- All of these points apply to any formal property of a logistic system and the represented property of English. For example, if a formal sentence is formally valid, it is usually required that the sentence of English be valid in the sense in which validity applies to English sentences. However, there are valid English sentences which no formally valid sentence of CQT adequately represents; e.g. "The apple is green" entails "The apple is coloured."

9. *The moral*? When we get right down to it, it all comes down to questions of the following sort:

- What is the good of knowing that there is a formal representation relation from a logistic system to a fragment of English in fulfillment of the condition that sentences A such that $\mathcal{V}$(A) = T pair with English sentences S such that S is true?
- Does this help us see what "true" means in English? Does it give us a more explicit and precise understanding of meaning in English?
- If so, in virtue of what? Consider logicism as an example. If we are worried about whether there is adequate reason to believe that "2 + 2 = 4" really is true, what good does it do us to know that there is a formal representation relation from the first-order functional calculus that pairs exactly one of its $\vDash$-sentences to "2 + 2 = 4"? After all, what "$\vDash$A" says is that every denumberably infinite sequence of abstract individuals of the domain of the system's interpretation satisfies A. Is any such thing true of "2 + 2 = 4"?

10. These are still open questions. They bear serious thinking about. It might not be a bad idea to keep in mind our *clarification spectrum*: ⟨analysis, explication, rational reconstruction, stipulation⟩.

- *analysis* makes the meaning of a concept (or of the term that expresses it) *explicit.*
- *explication* makes a meaning *precise.*
- *rational reconstruction* makes a meaning over.
- *stipulation* makes a new meaning up.

As we move from the spectrum's left terminus towards the one on the right, the further we progress from the original meaning (i.e. the subject-matter of the analysis). By the time we get to the terminus on the right, we have lost all contact with the original, and have changed the subject utterly (and usually on purpose).

The Tort List

grammar
syntax
semantics
vocabulary sentence (hence atomic sentence, also sentence-connectives)
formula
symbol
language
punctuator
axiom (hence schema of)
truth (hence logical truth, truth for an interpretation, truth-value, truth-table)
theorem
deduction
deducibility
proof (hence proof theory)
interpretation
tautology
sentence-validity
sequence-validity
entailment
demonstration
syntactic-consistency
semantic-consistency
name
predicate
discourse (hence domain of)
satisfaction
denotation
truth for an interpretation
falsity (hence falsity for an interpretation)